

## Lectures on Advanced Shell Elements for Composite Structures and Multifield Problems through the Sampling Surfaces Formulation

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**Venue** Sala Ferrari, DIMEAS

### Dates and Topics

16/11/2017, 11:30-13:00

Sampling Surfaces Formulation for Three-Dimensional Stress and Vibration Analyses of Layered and Functionally Graded Plates and Shells

23/11/2017, 14:30-16:00

Sampling Surfaces Formulation for Three-Dimensional Thermoelastic Analysis of Composite Plates and Shells

30/11/2017, 14:30-16:00

Hybrid-Mixed Geometrically Exact Elements for 3D Stress Analysis of Composite Plates and Shells through Sampling Surfaces Formulation

07/12/2017, 11:30-13:00

Finite Rotation Geometrically Exact Solid-Shell Elements for Laminated Composite Shells through Sampling Surfaces Formulation

The MUL<sup>2</sup> group in cooperation with AIDAA Torino and the FULLCOMP project is pleased to announce a series of lectures delivered by Professor Gennady Kulikov, currently a visiting Professor at DIMEAS.

### Abstract

It is well known that a conventional way of developing the higher-order shell formulation consists in the expansion of displacements into power series on the transverse coordinate referred to the direction normal to the middle surface. For the approximate presentation of the displacement field, it is possible to use finite segments of power series because the principal purpose of the shell theory consists in the derivation of approximate solutions of elasticity. The idea of this approach can be traced back to Cauchy and Poisson. However, its implementation for thick shells is not straightforward since it is necessary to retain a large number of terms in corresponding expansions to obtain the comprehensive results.

An alternative way of developing the shell theory is to choose inside each layer a set of not equally spaced sampling surfaces (SaS)  $\Omega^{(n)1}, \Omega^{(n)2}, \dots, \Omega^{(n)I_n}$  parallel to the middle surface in order to introduce the displacements, temperatures and electric potentials of these surfaces as basic shell variables, where  $I_n$  is the number of SaS of the  $n$ th layer ( $I_n \geq 3$ ) and  $n = 1, 2, \dots, N$ , where  $N$  is the total number of layers. Such choice of shell unknowns with the consequent use of Lagrange polynomials of degree  $I_n - 1$  in the thickness direction for each layer allows the presentation of governing equations of the SaS formulation in a very compact form. The feature of the SaS shell theory is that it is based on the strain-displacement relationships, which precisely represent all rigid-body motions of a shell in any convected curvilinear coordinate system. This fact has a great importance since one may read in the open literature that the shell theory is an absolute academic exercise due to the difficulties of representing the rigid body shell modes.

It is important that the SaS shell formulation with equally spaced SaS does not work properly with the Lagrange polynomials of high degree because of the Runge's phenomenon. This phenomenon can yield the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equispaced nodes is increased, then the oscillations become even larger. However, the use of the Chebyshev polynomial nodes inside the shell body can help to improve significantly the behavior of the Lagrange polynomials of high degree because such a choice permits one to minimize the error uniformly due to the Lagrange interpolation. Thus, the solutions based on the SaS technique asymptotically approach the 3D exact solutions of elasticity, thermoelasticity and electroelasticity as the number of SaS tend to infinity. This in turn gives an opportunity to derive the stress/strain state with a prescribed accuracy employing the sufficient number of SaS.

To avoid shear and membrane locking and have no spurious zero energy modes the hybrid-mixed GeX solid-shell element formulation can be utilized in which the SaS are located inside each layer at Chebyshev polynomial nodes. In the proposed hybrid-mixed four-node finite element formulation, the Hu-Washizu variational principle is invoked to introduce the assumed interpolations of displacements, displacement-dependent and displacement-independent strains and stress resultants. Such a finite element formulation for multifield problems exhibits excellent performance in the case of coarse mesh configurations and has computational advantages compared to conventional isoparametric hybrid-mixed solid-shell element formulations because it reduces the computational cost of numerical integration in the evaluation of the element stiffness matrix. This is due to the facts that all element matrices require only direct substitutions, i.e. no expensive numerical matrix inversion is needed. It is impossible in the framework of the isoparametric hybrid-mixed shell element formulation. Second, the GeX four-node solid-shell element formulation is based on the effective analytical integration throughout the finite element employing author's enhanced ANS method. The latter has great meaning for the numerical modelling of doubly-curved composite shell structures with variable curvatures



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### **Bio-Sketch**

Prof. Gennady M. Kulikov was born in Tambov on April 23, 1953. After earning a degree in Applied Mathematics and Mechanics (1975) at the Novosibirsk State University, he began working as a researcher at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences before joining Tambov State Technical University in 1981. He had been working for ten years as an Assistant Professor and an Associate Professor of the Department of Mathematics. From 1991 to 2015 he was a Professor and Head of the Department of Applied Mathematics and Mechanics. Since 2015, he is a Head of the Laboratory of Intelligent Materials and Structures. Prof. Kulikov received his first doctoral degree in Solid Mechanics in 1981 at the Moscow State University and his second degree of the Doctor of Physics and Mathematics in Solid Mechanics in 1991 at the Kazan State University. He was a Visiting Professor at the Institute of Mechanics, Technische Universität Berlin (1995, 1999, 2002) and the Department of Aeronautics and Aerospace Engineering, Politecnico di Torino (2007, 2016). He won international grants under INTAS/RFBR (1997-1999), DFG/RFBR (1998-2000) and RSF (2015-2017) foundations. He is the author of three books and more than hundred refereed journal articles on these topics published in international journals. He is a member of the editorial board of some international journals Journal of Applied Mathematics, Curved and Layered Structures and Coupled Systems Mechanics and many international conferences on computational mechanics and structural mechanics. He serves as a referee for more than twenty international journals.

For more info about these lectures, please contact Prof. Erasmo Carrera and Dr. Marco Petrolo ([erasmo.carrera@polito.it](mailto:erasmo.carrera@polito.it), [marco.petrolo@polito.it](mailto:marco.petrolo@polito.it), [www.mul2.com](http://www.mul2.com)).